



**Transportation Research Forum**

---

Optimizing Strategic Allocation of Vehicles for One-Way Car-sharing Systems Under Demand Uncertainty

Author(s): Wei (David) Fan

Source: *Journal of the Transportation Research Forum*, Vol. 53, No. 3 (Fall 2014), pp. 7-20

Published by: Transportation Research Forum

Stable URL: <http://www.trforum.org/journal>

---

The Transportation Research Forum, founded in 1958, is an independent, nonprofit organization of transportation professionals who conduct, use, and benefit from research. Its purpose is to provide an impartial meeting ground for carriers, shippers, government officials, consultants, university researchers, suppliers, and others seeking exchange of information and ideas related to both passenger and freight transportation. More information on the Transportation Research Forum can be found on the Web at [www.trforum.org](http://www.trforum.org).

**Disclaimer:** The facts, opinions, and conclusions set forth in this article contained herein are those of the author(s) and quotations should be so attributed. They do not necessarily represent the views and opinions of the Transportation Research Forum (TRF), nor can TRF assume any responsibility for the accuracy or validity of any of the information contained herein.

# Optimizing Strategic Allocation of Vehicles for One-Way Car-sharing Systems Under Demand Uncertainty

by Wei (David) Fan

*Car-sharing offers an environmentally sustainable, socially responsible and economically feasible mobility form in which a fleet of shared-use vehicles in a number of locations can be accessed and used by many people on as-needed basis at an hourly or mileage rate. To ensure its sustainability, car-sharing operators must be able to effectively manage dynamic and uncertain demands, and make the best decisions on strategic vehicle allocation and operational vehicle reallocation both in time and space to improve their profits while keeping costs under control. This paper develops a stochastic optimization method to optimize strategic allocation of vehicles for one-way car-sharing systems under demand uncertainty. A multi-stage stochastic linear programming model is developed and solved for use in the context of car-sharing. A seven-stage experimental network study is conducted. Numerical results and computational insights are discussed.*

## INTRODUCTION

A successful economy relies largely upon an efficient, effective and sustainable transportation system. In many urban settings worldwide, this is increasingly highlighted, and new transportation mobility solutions are constantly being developed to accommodate increasing demands on urban infrastructure. Many Americans have realized that the price of gasoline has been increasing for years. From 2010 to 2011, this increase was 26.4% (U.S. Department of Labor 2011) and led to the largest increase of 8.0% in total transportation cost in an economy which saw a meager 1.9% growth in average annual income. In total, transportation was second only to housing and accounted for 13% of total spending (U.S. Department of Labor 2012). Though transportation's cost is increasing, many people in large urban areas do not need to fully own a vehicle because ownership invokes the expense of purchasing, licensing, insuring, and parking, which may not be justified by the little traveling they do. Some countries (e.g., Germany and Canada) have realized this and have embraced the idea of car-sharing as a short-term auto use mobility solution (Shaheen and Cohen 2007). This allows customers to use a vehicle only when they need it. Undoubtedly, this program can help alleviate urban congestion and parking issues in many cities. In particular, to create a successful car-sharing program, car-sharing operators must be able to effectively manage dynamic and uncertain demands, and make good decisions on strategic vehicle allocation and operational vehicle reallocation both in time and space to improve their profits while keeping costs under control.

The problem of optimizing strategic allocation of car-sharing vehicles (OSACV) addressed in this paper can be presented as follows. Car-sharing operators must be able to determine the most efficient means of strategically allocating vehicles at multiple car-sharing locations to accommodate future uncertain demands to maximize total expected profits. There must be a way to determine if any demand is unprofitable so as to refuse it and use resources efficiently to achieve a more favorable vehicle reallocation in the future. OSACV is a very complex stochastic optimization problem due to increasing levels of uncertainty associated with future demands. It requires car-sharing operators to not only make optimal decisions as to strategic allocation of vehicles and operational reallocation of vehicles given strategic vehicle positions, but also to anticipate the impact of such decisions in future periods.

Despite many relevant studies, no research solves the OSACV problem. Additionally, all previous studies assumed that decisions on strategic allocations of car-sharing vehicles had been made and that the vehicle supply on the first day (i.e., the initial allocation at each business location at the start of operations) was already known. In the real world, however, to maximize long-term profits, car-sharing operators must first make the best decision in terms of how to strategically allocate vehicles in space (i.e., a network of locations) to ensure they are well distributed and optimally positioned to accommodate future uncertain demands (in both time and space). After that, car-sharing operators operationally reallocate/optimize vehicles in both time (e.g., on a daily basis) and space to improve their revenues while keeping costs under control. In that regard, there is a strong need to optimize strategic allocation of car-sharing vehicles because it can have significant impacts on later car-sharing operations, which may require a large number of used vehicle movements and empty vehicle relocations to balance the fleet across locations and days based on this strategic allocation decision/input.

As such, the purpose of this paper is to develop a stochastic optimization approach to solve the OSACV problem, which is not discussed in previous studies. A multi-stage stochastic linear programming model is designed to optimize strategic allocation of car-sharing vehicles in space under demand uncertainty and is validated based on a pilot study. This validation gives results that are unique because there has never before been a study to solve the OSACV problem using stochastic programming approaches instead of simulation-based models.

The rest of this paper is organized as follows: The second section is a review of existing literature. The third presents the methodology, which includes assumptions made and stochastic linear programming model formulation for OSACV. The fourth section illustrates a scenario tree based stochastic programming solution along with the scenario tree generation, and section five describes the experimental network used in the pilot study. The sixth section discusses the computational results while the final section summarizes and discusses the results as well as provides directions for future research.

## LITERATURE REVIEW

### General Review of Car-sharing

Car-sharing was introduced to Switzerland in 1987, and Germany soon joined in 1988. It wasn't until 1993 that it came to North America when Quebec, Canada, created a car-sharing program. In the past 20 years, car-sharing has grown across the world (Shaheen and Cohen 2007, Shaheen et al. 2006, Carsharing 2013a, Carsharing 2013b). As of October 2012, it was present in more than 27 countries and five continents with approximately 1,788,000 individuals sharing over 43,550 vehicles (Carsharing 2013b). North America is home to 45 car-sharing programs with 26 in America and 19 in Canada. The United States has approximately 806,332 car-sharing members (Carsharing 2013b). Examples of these programs can be seen in Austin TX, Chicago IL, New York City NY, Philadelphia PA, San Francisco CA, Seattle WA, and Washington DC (Carsharing 2013a, Carsharing 2013b).

The principle of car-sharing is very simple: "Individuals gain the benefits of private vehicle use without the costs and responsibilities of ownership" (Shaheen and Cohen 2007, Shaheen et al. 2006). A car-sharing member, business owner, or household can access a fleet of shared-use vehicles, which are located in a network of locations and are maintained by the organization that runs the car-sharing program (Shaheen and Cohen 2007, Shaheen et al. 2006). To participate, a customer purchases a membership key or card and makes an appointment by phone or Internet to use a vehicle in the fleet. Once approved, the vehicle is made available to a client who picks it up at an appointed time and leaves it at a designated car-sharing location, which may be the same as the pick-up point (one-way car-sharing systems) or anywhere in a specified zone (free-floating car-sharing systems). The customer is charged a user fee but not a maintenance fee, which is borne by the car-sharing company. In all, this program gives many people access to a fleet of shared-use vehicles without owning them.

Car-sharing has many benefits, including reduced personal transportation costs because customers only pay user fees, and it results in fewer vehicle trips, which in turn reduce traffic congestion. Other advantages are that it uses fuel efficient vehicles and in so doing, reduces fuel use and emissions, and improves roadway safety because it results in fewer vehicle miles traveled. Also, rational urban development patterns with efficient land use can be achieved because it results in fewer vehicles per capita and fewer parking locations, and it allows easy coordination of different modes of transportation, especially when their locations are near bus routes and rail stations. Finally, car-sharing provides lower income households with increased mobility by giving them the option of personal vehicle use without the expense of its ownership.

### **Car-sharing Fleet Management**

During the past 20 years, the feasibility, operation, and safety of car-sharing have been comprehensively studied. For example, Shaheen and Cohen (2007) compared car-sharing in different countries in terms of member-vehicle ratios, market segments, parking approaches, vehicles and fuel, insurance, and technology. Their research findings are summarized below. Germany, Switzerland, and the United States distinguished themselves from their international counterparts with higher member-vehicle ratios largely due to market diversification and less active users in the United States and Germany, and inactive members in Switzerland. On-street parking in most car-sharing countries (France, and Spain) was a common form of public non-monetary operator support. And, although there were distinct regional differences in alternative fuel vehicle use, conventional gasoline automobiles accounted for most of the fleets (except in Japan and Spain).

Barth and Todd (1999) simulated car-sharing programs that included the ability to calculate vehicle availability, vehicle distribution, and energy management. They applied this to a resort community in Southern California and found that the shared vehicle system was most sensitive to the vehicle-to-trip ratio, the relocation algorithm used, and the charging scheme employed. Their preliminary cost analysis indicated that such a system could be very competitive with present transportation systems (e.g., rental cars, taxis, etc.). Kek et al. (2006) also used simulation to investigate car-sharing by emphasizing operator-based relocation techniques. They were able to help operators maximize efficiency and increase service levels and validated their model using data collected by an operational car-sharing company in Singapore.

In later work, Kek et al. (2009) presented a simulation-based decision support system to determine a set of near-optimal manpower and operating parameters for vehicle relocation operations in car-sharing systems. They tested their approach in Singapore and reported that it resulted in a 50% reduction in staff cost, a reduction in zero-vehicle-time (i.e., time when stations have no vehicles available) ranging between 4.6% and 13.0%, a maintenance of the already low full-port-time, and a 37.1%-41.1% reduction in number of relocations. Nair and Miller-Hooks (2011) and Nair et al. (2013) presented some interesting optimization work for vehicle and bike sharing, respectively. In a real-world application of their work to a system in Singapore, they found that fleet management strategies which explicitly accounted for the stochastic nature of demand offered greater reliability than strategies based on static methods. Also, fleet redistribution strategies based on their approach were better than those from scenarios in which demand outstripped supply. Despite these studies, few have applied stochastic programming models for car-sharing fleet management (by optimizing strategic vehicle allocation), though such an approach has been widely used in general fleet management research.

### **Review of Stochastic Programming and its Applications in Fleet Management**

The stochastic dynamic vehicle allocation problem (SDVAP) for car-sharing has been studied in the past because it is common in freight and other transportation industries. Some of these studies involve static deterministic, static stochastic, and dynamic deterministic formulations as well as

dynamic vehicle allocations under uncertainty and potentially infinite time horizons. However, very few studies have been done on dynamic stochastic formulations of the car-sharing problem. Among them, Dejax and Crainic (1987) presented a taxonomy of empty vehicle flow problems and models and conducted a comprehensive review of the existing literature on the subject. Major research trends and perspectives were identified, and the advantages of a hierarchically integrated approach for simultaneous management of empty and loaded freight vehicle movements were also discussed. Jordan and Turnquist (1983) discussed uncertainties in vehicle supply and demand and proposed a system for railroad freight cars to optimize their allocations, which was solved with the Frank-Wolfe algorithm. This iterative algorithm uses linear approximations to a nonlinear objective function and solves it as linear programming sub-problems until it converges to an optimal solution for a nonlinear (possibly concave) objective function.

Powell (1986) and Fantzeskakis and Powell (1990) used stochastic formulation of SDVAP and proposed a heuristic algorithm which contrasted various deterministic approximations. This algorithm used a rolling horizon procedure to simulate the operations of railroads and truck carriers. They conducted experiments for a 12-day period for different fleet sizes, and their numerical results indicated the superiority of their algorithm to other approaches they tested in terms of total profit. Bookbinder and Sethi (1980), Cheung and Chen (1998), Cheung and Powell (2000), and Fan and Machemehl (2007) studied SDVAP using dynamic stochastic formulations and suggested that they had advantages over their counterparts and should be studied further.

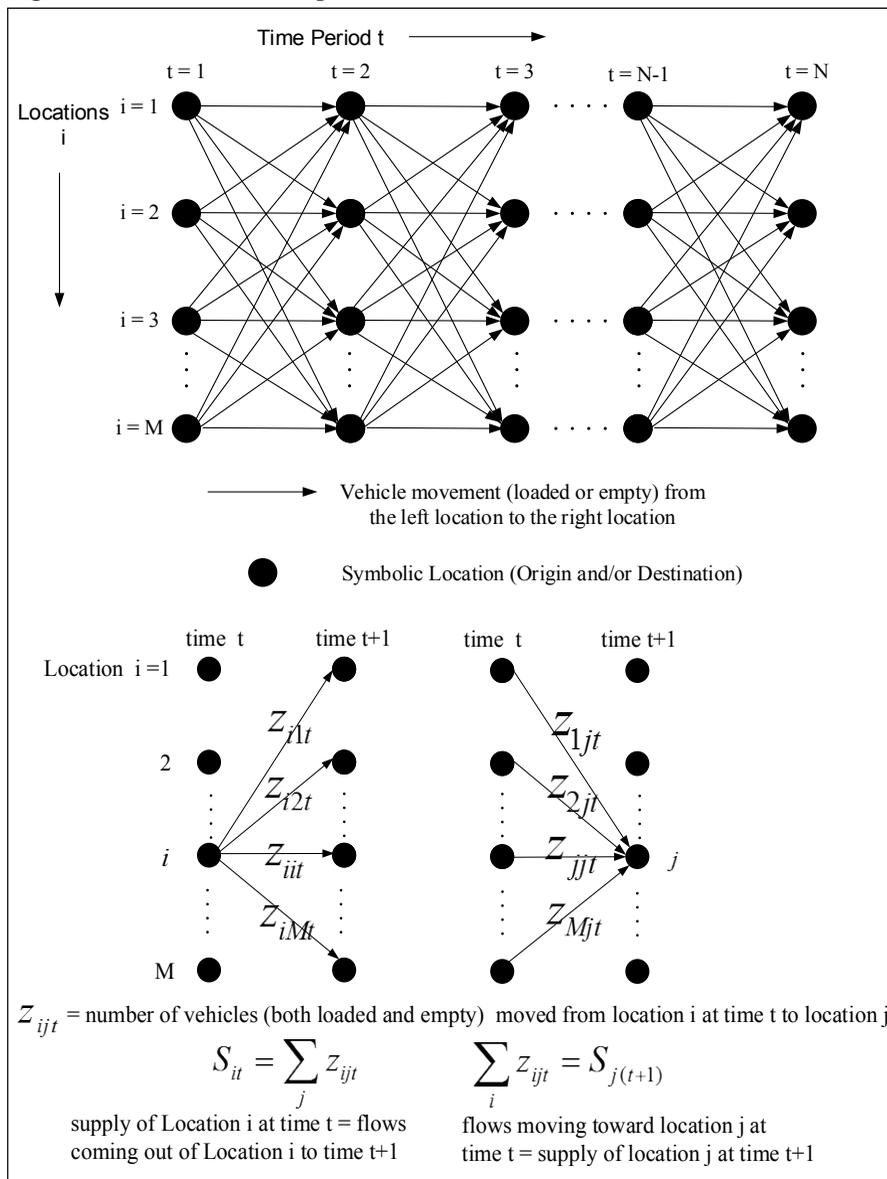
Although these previous studies are very helpful, little has been done on using stochastic optimization techniques managing and operating car-sharing programs. An exception is Fan et al. (2008), who studied SDVAP to maximize profits for a car-sharing service operator. To do this, developed a multi-stage stochastic linear integer model and solved it with a Monte Carlo sampling-based stochastic optimization method in which Monte Carlo simulation was used to realize uncertain demands that were assumed to be Poisson distributed. The car-sharing dynamic vehicle allocation problem was solved and fleet management was optimized in both time and space. As a pilot study, a five-stage example network with four car-sharing locations was designed to test the developed method. The computational results indicated a high-quality SDVAP solution, suggesting that the algorithm could be used for real-world applications. Fan (2013) later developed a stochastic optimization framework to address SDVAP for car-sharing systems. Rather than using Monte Carlo sampling methods, he assumed uncertain demands to be discretely distributed, generated a complete scenario-tree, and solved it using stochastic optimization techniques. The computational results indicated a high-quality solution, suggesting that stochastic optimization can be used in real-world applications. Of note is that both studies dealt with the dynamic vehicle allocation on a day-to-day operational level, and assumed that strategic vehicle allocation (i.e., the vehicle supply) across a car-sharing network was given. Also, both studies were based on large-scale linear and multi-stage stochastic programming theory found in Dantzig (1955), Dantzig and Wolfe (1960), Ziemba (1970), Wollmer (1980), Wets (1983), Birge (1985), Birge and Louveaux (1997), Wallace (1986), and Beale et al. (1986). Scenario trees are important parts of this technique, and Zenios (1998), Kouwenberg (2001), Hoyland and Wallace (2001), and Fleten et al. (2002) developed such trees for multi-stage stochastic programming decision problem scenarios.

## **METHODOLOGY**

To study the OSACV problem, several assumptions are made in this paper. Some are that customers reserve vehicles at the end of each day, specific pickup locations are used by customers who can drop off cars at any specified (it can be the same or different) location at a specific time (one day after pick up), and one vehicle is allocated per demand per customer. The vehicles are in use, in transit empty or stationary empty, and the travel time between all car-sharing locations is one day, whether a vehicle is in use or empty. Though no future information is available to car-sharing managers, the expected (i.e., mean) demand at the beginning of each day throughout the decision horizon (when the strategic allocation of vehicles decision is to be made) is always known. Furthermore, it

is assumed that though the mean values and probability distribution of uncertain demands between all pairs of car-sharing locations are not necessarily equal, they are known and discretely distributed (e.g., can be classified into and labeled as HIGH, MEDIUM, and LOW demand scenarios). And, it is assumed that on all days of operation, every vehicle is available for use, that the expected (i.e., mean) demands between all pairs of car-sharing locations at all times can be forecasted and determined based on relevant market surveys, and the demands on different days may be independent of each other, or can be correlated. Nonetheless, all daily car-sharing demand forecasts are assumed to be known and used as inputs to the OSACV model, and future vehicle availability is directly affected by current strategic allocation decisions on vehicle use. From these assumptions, the formulation of OSACV and the graphs of the network studied are in Figure 1. The profit from servicing demand is the difference between the revenue collected and the cost incurred. Also, “flow of vehicles” and the “number of vehicles” are equivalent as each vehicle only carries one passenger.

**Figure 1: Network Flow Representation of the OSACV**



**Model Formulation**

Based on the assumptions, a stochastic programming formulation of the OSACV problem is in Table 1 for a planning horizon of  $N$  days.

**Table 1: Definition of Terms**

<b>Indices/Sets</b>	
$i, j \in R$	Regional carsharing pick-up origins and/or drop-off destinations
$t \in T$	Time periods
$w \in W$	Demand scenarios, in which each demand scenario consists of a complete realization set of specific demands at each stage
<b>Random Variables</b>	
$\tilde{d}_{ijt}$	Random demand denoting the number of customers needing transportation from location $i$ to location $j$ during period $t$ , $t = 2, \dots, N$
<b>Parameters/Data</b>	
$r_{ij}$	Net revenue for satisfying a carsharing demand from pickup location $i$ to dropoff location $j$
$c_{ij}$	transfer cost of moving empty from location $i$ to location $j$
$L_{ijt}$	Number of carsharing requests known at time $t = 1$ to be available moving from location $i$ to location $j$ at the first time period during the current planning horizon
$d_{ijt}^w$	Demand denoting the number of customers needing transportation from location $i$ to location $j$ during period $t$ under demand scenario $w$ , $t = 2, \dots, N$
$p^w$	Probability of demand scenario $w \in W$
<b>Decision Variables</b>	
$x_{ijt}^w$	Number of carsharing vehicles that are used by customers from location $i$ to location $j$ , during period $t$ under demand scenario $w$ , $t = 1, 2, \dots, N$
$y_{ijt}^w$	Number of carsharing vehicles moving empty from location $i$ to $j$ , during period $t$ under demand scenario $w$ , $t = 1, 2, \dots, N$
$Z_{ijt}^w = x_{ijt}^w + y_{ijt}^w$	Number of carsharing vehicles moving loaded or empty from location $i$ to location $j$ , during period $t$ under demand scenario $w$ , $t = 1, 2, \dots, N$
$S_{i1}$	Strategic allocation/supply of carsharing vehicles at location $i$ at the beginning of period 1
$S_{it}^w$	Supply of carsharing vehicles at location $i$ at the beginning of period $t$ under demand scenario $w$ , $t = 2, \dots, N$

**Objective function:** The optimization model for the strategic allocation of car-sharing decision-making problem for each period  $t = 1, 2, \dots, N$  can be presented as follows:

$$\begin{aligned}
 (1) \quad & h(S_{i1}, x_{ijt}^w, y_{ijt}^w, d_{ijt}^w) = \max_{S_{i1}, x_{ijt}^w, y_{ijt}^w} \sum_i \sum_j \sum_t \sum_w (r_{ij} x_{ijt}^w - c_{ij} y_{ijt}^w) * P^w \\
 (1a) \quad & \text{subject to: } x_{ij1}^w \leq L_{ij1} \quad i \in R, j \in R, w \in W \\
 (1b) \quad & x_{ijt}^w \leq d_{ijt}^w \quad i \in R, j \in R, t = 2, 3, \dots, N, w \in W \\
 (1c) \quad & \sum_j (x_{ijt}^w + y_{ijt}^w) = S_{it}^w \quad i \in R, t = 2, 3, \dots, N, w \in W \\
 (1d) \quad & \sum_j (x_{ij1}^w + y_{ij1}^w) = S_{i1} \quad i \in R, w \in W \\
 (1e) \quad & \sum_i (x_{ijt}^w + y_{ijt}^w) = S_{j(t+1)}^w \quad j \in R, t = 1, 2, \dots, N-1 \\
 (1f) \quad & x_{ijt}^w, y_{ijt}^w, S_{it}^w \geq 0 \quad i \in R, j \in R, t = 1, 2, \dots, N, w \in W \\
 (1g) \quad & x_{ijt}^w, y_{ijt}^w, S_{it}^w \text{ must be integers } \quad i \in R, j \in R, t = 1, 2, \dots, N, w \in W
 \end{aligned}$$

As can be seen, the objective function is to maximize total expected profits. The cost of unmet demand is not considered but can be easily incorporated if desired by including a penalty in the objective function. Constraints (1a) and (1b) state that all loaded movements serving the transportation needs of car-sharing customers must be less than or equal to the requested demand for those movements under each scenario for all periods. Constraints (1c) and (1d) indicate that the total number of loaded and empty vehicles that move out of a location  $i$  at any period  $t$  must be equal to the total vehicles available at location  $i$  during all future periods under each scenario, or the number of strategically allocated vehicles during the first period. Constraints (1e) represent flow conservation properties, which guarantee that the number of vehicles available at location  $j$  at period  $t + 1$  must be equal to the number of loaded or empty movements to location  $j$  during period  $t$  under each scenario. Constraints (1f) and (1g) represent non-negativity and integer properties, respectively. In particular, for OSACV, the strategic allocation/supply of car-sharing vehicles at location  $i$  at the beginning of period one is a decision variable which can be optimally solved and determined by the model as presented above. Once such a strategic vehicle allocation decision has been made, the number of vehicles allocated across all locations in the car-sharing network is used as an input to determine the optimal car-sharing vehicle reallocation pattern at the operational (e.g., day-to-day) level for all future periods.

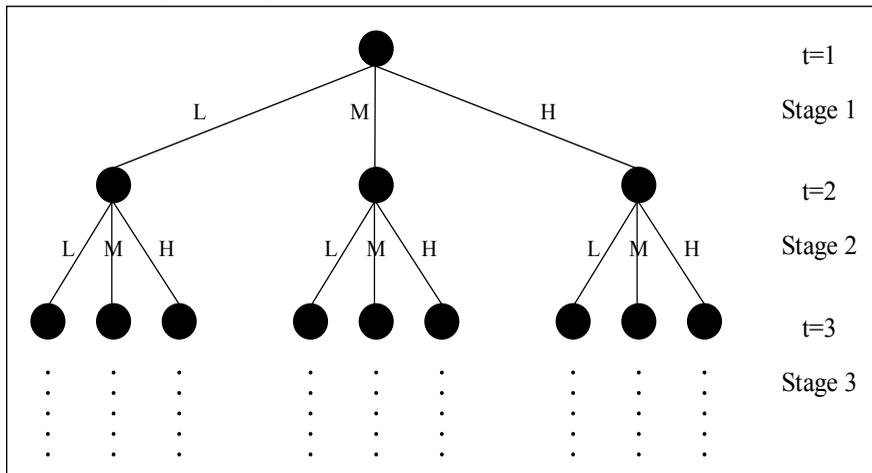
Problems involving uncertainty in the objective function and/or constraints fall in the domain of stochastic programming. Furthermore, it can be seen that OSACV is a linear programming problem with uncertain constraint coefficients on the right-hand side. The major difficulty arising from this problem is the required truncation of their infinite planning horizon to a finite number of periods ( $N$ ), which might cause deviations from the infinite planning horizon's optimal solution. Because of this, OSACV can be treated as a multiple-stage (here  $N$ -stage) Stochastic Linear Programming (SLP) problem in which a decision made in the following stage can compensate for any bad effects that might have been experienced as a result of the previous-stage decision. Several approaches have been proposed to solve multiple-stage SLP. When the problem size is manageable, the simplex, interior point and decomposition methods are generally very efficient in solving it (Birge and Louveaux 1997).

## SOLUTION APPROACH

Figure 2 illustrates a complete scenario-tree-based approach for solving the multi-stage stochastic programming models. The nodes in the tree represent states at a particular period,  $t$ . Decisions are made at the nodes, and the arcs represent realizations of uncertain variables. Decisions to be made further down the scenario tree depend on those already made through parent nodes and the uncertain properties of descendant nodes, such as the three L(ow), M(edium), and H(igh) car-sharing demand scenarios shown. The generation of the scenarios is based on the assumed discrete distribution, and decision makers can specify the probability distribution function so that the statistical properties of the problem are preserved. A complete scenario tree consists of realizations of the uncertain variables of each period (or stage). In practice, only the first-stage solution at the top node is used for decision making. Decisions at stage two or after are only done to find the right incentives for the first-stage decisions (Fleten et al. 2002).

At the beginning of the first period, decisions are made based on current information (and realizations of the stochastic future) and at the end, the effects of these decisions are seen. Given these effects and new information for the next period, a new decision is made. Based on the consequences from the second period decisions and given new information for the third, a decision is made again. The whole process continues indefinitely in principle. For each scenario tree with generated random variates, one can use exact optimization methods (e.g., L-shaped Method – see Birge and Louveaux 1997) to solve it. In fact, the first-stage decision is obtained by developing and running an SAS macro-based SAS/OR PROC OPTMODEL code (SAS 2011).

**Figure 2: Scenario Tree Approach for Solving the Multi-stage Stochastic Programming Models**



## EXPERIMENTAL DESIGN

A car-sharing dynamic vehicle allocation problem represented by a seven-stage experimental network (specifically, a planning horizon of seven days and four car-sharing locations) was designed as a pilot study to test the quality and efficiency of the solution using the developed stochastic programming method to solve OSACV. As presented in the model formulation section, the demands at each car-sharing location on the first day are assumed to be known with certainty, and the expected (i.e., mean) values of the uncertain car-sharing demands on days two to seven are assumed to be known and used as input to OSACV. As mentioned, all stochastic demands on days two to seven are assumed to follow a discrete distribution, which constitutes three-level (HIGH, MEDIUM, and LOW) demand scenarios. All the supply and demands are expressed as units of “vehicles.” The net

revenue per loaded vehicle and the cost per empty vehicle are expressed as matrices in dollars. The input required for the OSACV example is summarized in Table 2.

To solve OSACV, the expected value of the uncertain demand (i.e., the mean demand as shown in Table 2) is used as the known initial demand on the first day. This is deemed reasonable because car-sharing operators only need the mean demand to make strategic vehicle allocation decisions. However, once such a strategic vehicle allocation decision has been made, the number of vehicles allocated across locations in the car-sharing network is used as an input to determine the optimal car-sharing vehicle reallocation pattern at the operational (e.g., day-to-day) level for all future periods.

**Table 2: Input to the OSACV Example**

Mean Demand for All Following Days (Unit: Vehicles)		Drop-off Destination			
		1	2	3	4
Pick-up origin	1	11	8	7	15
	2	5	9	12	8
	3	10	12	11	7
	4	10	17	10	16

Stochastic High Demand (Prob=0.4) (Unit: Vehicles)		Drop-off Destination			
		1	2	3	4
Pick-up origin	1	16	14	10	22
	2	8	14	18	12
	3	16	18	15	11
	4	15	24	15	25

Stochastic Medium Demand (Prob=0.2) (Unit: Vehicles)		Drop-off Destination			
		1	2	3	4
Pick-up origin	1	13	6	7	15
	2	5	9	12	10
	3	10	12	11	9
	4	10	15	10	14

Stochastic Low Demand (Prob=0.4) (Unit: Vehicles)		Drop-off Destination			
		1	2	3	4
Pick-up origin	1	5	3	4	8
	2	2	4	6	3
	3	4	6	7	2
	4	5	11	5	8

Net Revenue (Unit: \$)		Drop-off Destination			
		1	2	3	4
Pick-up Origin	1	8	12	19	15
	2	10	11	18	17
	3	14	16	9	19
	4	15	17	19	12

Transfer Cost (Unit: \$)		Drop-off Destination			
		1	2	3	4
Pick-up Origin	1	0	3	4	4
	2	3	0	4	5
	3	4	4	0	4
	4	4	5	4	0

Initial Total Supply (Unit: Vehicles)	171
--	-----

The strategic car-sharing vehicle allocation decision-making problem in this example is a typical multi-stage stochastic programming problem. Although it involves dynamic vehicle allocations in only seven periods and at only four car-sharing locations, it is not a minor problem. For example, assume a three-level case in which the demand at each stage has three associated situations, namely HIGH, MEDIUM, and LOW. Then, the total number of decision variables involved in all scenarios of this problem is the sum of  $(3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6) * 4^2$  loaded vehicle movement decisions,  $(3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6) * 4^2$  empty vehicle movement decisions, four strategic vehicle allocation decisions and  $(3^1 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6) * 4^1$  operational vehicle reallocation decisions, which are 39,348 decisions altogether.

**RESULTS AND DISCUSSION**

**Strategic Allocation of Car-sharing Vehicles**

**Stochastic Programming Approach.** The stochastic programming (SP) solution has been developed and had its quality tested. As mentioned, SP is a modeling framework for handling uncertainty in some of the problem data (e.g., the stochastic demand in this paper). Once a scenario tree has been set up with car-sharing demands between origin-destination pairs with three levels (i.e., low, medium, and high) during all periods, an exact stochastic optimization method (e.g., L-shaped method) is used to solve the OSACV problem and obtain the first-stage decision. Since future car-sharing demands are stochastic (although with some known discrete probabilities) and a decision must be made at the current period (i.e., Stage 1), the values of all first-period decision variables must be the same for all scenarios. Using the developed SAS macro code, the solution obtained for the three-level case shown in Table 3 is SP of \$14,664 for all possible scenarios. This value means that one can earn \$14,664 on average if such SP strategic vehicle allocation decision is used.

**Table 3: Numerical Results**

	Solution Approaches	Solution Descriptions	Objective Function Values
Developing Solutions	Deterministic Optimization	Expected Demand (ED) (i.e., Optimize the average scenario)	\$16,460
		Wait-and-See (WS)	\$14,718
	Stochastic Programming	Stochastic Programming (SP) under Three Scenarios: HIGH, MEDIUM and LOW	\$14,664
Processing Solutions	Evaluating Solutions under Stochastic Environment	Evaluating ED (EED)	\$14,641
Comparing Solutions	Value of Perfect Information (VPI)	$VPI = WS - SP$	\$54
	Value of Stochastic Solution (VSS)	$VSS = SP - EED$	\$23

### Deterministic Optimization Approach

Deterministic optimization (DO) solutions, including the expected demand (ED) solution and wait-and-see (WS) solution, were also developed.

**Expected Demand Solution.** It is common to ignore the uncertainties associated with system parameters because of computational inconveniences they cause and instead develop heuristic decisions using the expected value of the random variables. In other words, car-sharing operators may make the decision to calculate the weighted average demand of the three scenarios as shown in the above example and then execute the optimal solution by optimizing the “average” scenario. By doing so, the OSACV problem is solved by replacing random demands with their expected values and the solution in Table 3 shows a profit of \$16,460. This is expected because the problem has changed from stochastic programming to deterministic optimization. When demand is deterministic instead of random, one has certain demand information and, as a result, one can get a better solution compared with the stochastic programming approach with an objective function value of SP equal to \$14,664. Also, the value of the objective function for the expected value problem is greater than that of the stochastic problem (for this profit maximization OSACV problem), which accords with Jensen’s inequality principle (Birge and Louveaux 1997) commonly used in the trucking industry, where first-stage decisions are usually made based only on expected demand.

**Wait-and-See Solution.** Assuming that perfect information can be obtained, that is, stochastic demands are now deterministically known for all origin and destination pairs throughout the entire optimization period for all scenarios, then OSACV changes to a deterministic model (sometimes called the wait-and-see strategy) whose solution provides an upper bound for the optimal SP for maximization problem (Birge and Louveaux 1997). By solving several separate stochastic linear programming models, each consisting of a separate (unrelated) scenario tree with realized demand information, the wait-and-see (WS) solution results as shown in Table 3 is a profit of \$14,718, which is greater than that of the stochastic problem. This is expected because one can always earn more if perfect information about demand at each stage is known. Furthermore, the WS provides a tighter upper bound than the expected demand problem, which is predicted by the theorem that  $WS \leq ED$  (again for the maximization problem) if only right-hand-side variables are random (Birge and Louveaux 1997).

### Comparing SP and DO Solutions

**Comparing Decision Results.** The optimal solution developed using the SP approach is as follows: 41 vehicles should be allocated to location 1, 34 to 2, 40 to 3, and 56 to 4. On the other hand, the DO-ED approach gives a different solution: 41 and 40 vehicles are still allocated to locations 1 and 3, respectively. However, 30 vehicles are allocated to location 2 and 60 vehicles to 4. These

clearly show that different solutions would be obtained if different approaches are used to solve the OSACV problem.

In addition, the recommended vehicle allocation with the first day's demands by both solution approaches show that some demands are refused and some vehicles are assigned to move empty. This is expected because the goal to maximize profits results in unprofitable demands (with lower profit) not being satisfied and spare empty vehicles allocated to a more favorable future location. While this may not be exactly parallel to current car-sharing management practices (in which all car-sharing demands are typically served even when some are not profitable), the results certainly can be extended to a real-world car-sharing management system.

**Value of Perfect Information.** Executing the SP solution puts the car-sharing company in the best position to handle any demand scenario that might occur in the future. However, if the future can be forecasted, then the car-sharing company could optimize its solution separately for any of the scenarios in the wait-and-see solution. The difference between the deterministic “wait-and-see” and the SP solutions is commonly known as the Value of Perfect Information (VPI) (Birge and Louveaux 1997). In this regard, VPI can be calculated as  $VPI = WS - SP = \$54$ , which is the total amount a car-sharing fleet manager is willing to pay to obtain perfect information on system demands for all periods.

**Value of Stochastic Solution.** The difference between the SP solution's cost and the total cost of using the “expected demand” solution (where the solution for the expected demand case is used as the “average” scenario solution and evaluated under stochastic environment) corresponds to the Value of Stochastic Solution (VSS). In other words, if the solution of the expected demand problem is evaluated in the random demand environment, the objective function of the stochastic problem becomes EED of \$14,641, which is actually worse than the SP solution ( $SP = \$14,664$ ). That is, VSS can be calculated as  $VSS = SP - EED = \$23$ , which can be explained as the cost of ignoring system demand uncertainty and always using their expected values instead. Although this amount might not seem large, the aggregate value for a large network under a long time horizon can be significant. Therefore, stochastic solutions are always preferable to expected demand solutions.

**Computation Efficiency.** As described above, the SP approach has been used to solve the OSACV problem in this paper. One of its promising properties is that execution time is very short. It takes only about 10 seconds for the SAS code execution of the example network. Furthermore, as previously shown, the computational quality of the solution is very good. These two characteristics strongly suggest that this stochastic programming approach could be applied to solve the OSACV problem efficiently and effectively.

## CONCLUSIONS AND FUTURE RESEARCH

Car-sharing offers an environmentally sustainable, socially responsible, and economically feasible mobility form in which a fleet of shared-use vehicles in a number of locations can be accessed and used by many people rather than a single owner on an as-needed basis at an hourly or mileage rate. It allows users to enjoy the benefits of having personal vehicles but without the responsibilities and costs of ownership. This paper develops a stochastic optimization approach to solve the optimal strategic allocation of vehicles for one-way car-sharing systems, in which operators must be able to effectively manage dynamic and uncertain demands, strategically make the best decision in allocating vehicles, and operationally optimize vehicle reallocation both in time and space to improve their revenues while keeping costs under control. A multi-stage stochastic linear programming model with recourse is created and solved for use in car-sharing under demand uncertainty. A seven-stage example network with four car-sharing locations is designed to test the SP approach. The computational results indicate high quality OSACV solutions, suggesting that the SP algorithm can be used for real-world applications. Further research validating the SP formulation for the OSACV

problem using real-world large networks will be useful. In addition, archived historical data can be used to validate it and additional constraints may be incorporated in the optimization model.

## References

- Barth, Matthew and Michael Todd. "Simulation Model Performance Analysis of a Multiple Station Shared Vehicle System." *Transportation Research Part C: Emerging Technologies* 7(4), (1999): 237–259.
- Beale, E.M., George B. Dantzig, and R.D. Watson. "A First Order Approach to a Class of Multi-time Period Stochastic Programming Problems." *Mathematical Program Study* 27, (1986): 103-177.
- Birge, John R. "Decomposition and Partitioning Techniques for Multistage Stochastic Linear Programs." *Operations Research* 33, (1985): 989-1007.
- Birge, John R. and François Louveaux. *Introduction to Stochastic Programming*. Springer Series in Operations Research, New York, 1997.
- Bookbinder, James H. and Suresh P. Sethi. "The Dynamic Transportation Problem: a Survey." *Naval Research Logistic Quarterly* 27, (1980): 447-452.
- Carsharing. 2013a. <http://www.carsharing.net>, Accessed April 2, 2014.
- Carsharing. 2013b. <http://www.innovativemobility.org/carsharing/index.shtml>, Accessed April 2, 2014.
- Cheung, Raymond K. and Chuen-Yih Chen. "A Two-stage Stochastic Network Model and Solution Methods for Dynamic Empty Container Allocation Problem." *Transportation Science* 32(2), (1998): 142-162.
- Cheung, Raymond K. and Warren Powell. "SHAPE: A Stochastic Hybrid Approximation Procedure for Two-stage Stochastic Programs." *Operations Research* 48(1), (2000): 73-79.
- Dantzig, George. "Linear Programming Under Uncertainty." *Management Science* 1, (1955): 197-206.
- Dantzig, George and Philip Wolfe. "Decomposition Principle for Linear Programs." *Operations Research* 8, (1960): 101-111.
- Dejax, Pierre J. and Teodor G. Crainic. "A Review of Empty Flows and Fleet Management Models in Freight Transportation." *Transportation Science* 21, (1987): 227-247.
- Fan, Wei and Randy B. Machemehl. "A Multi-stage Monte Carlo Sampling Based Stochastic Programming Model for the Dynamic Vehicle Allocation Problem." *Journal of Advances in Transportation Studies* 12, (2007): 27-44.
- Fan, Wei, Randy B. Machemehl, and N. Lowmes. "Carsharing: Dynamic Decision-Making Problem for Vehicle Allocation." *Transportation Research Record: Journal of the Transportation Research Board* 2063, (2008): 97-104.
- Fan, Wei. "Managing Dynamic Vehicle Allocation for Carsharing Systems: Stochastic Programming Approach." *Transportation Research Record – Journal of Transportation Research Board* 2359, (2013): 51-58.

- Fleten, Stein-Erik, Kjetil Hoyland, and S.W. Wallace. "The Performance of Stochastic Dynamic and Fixed Mix Portfolio Models." *European Journal of Operational Research* 140, (2002): 37-49.
- Frantzeskakis, Linos F. and Warren B. Powell. "A Successive Linear Approximation Procedure for Stochastic Dynamic Vehicle Allocation Problems." *Transportation Science* 24, (1990): 40-57.
- Hoyland, Kjetil and Stein W. Wallace. "Generating Scenario Trees for Multistage Decision Problems." *Management Science* 47(2), (2001): 295-307.
- Jordan, William C. and Mark A. Turnquist. "A Stochastic Dynamic Model for Railroad Car Distribution." *Transportation Science* 17, (1983): 123-145.
- Kek, Alvina G.H., Ruey L. Cheu, and M.L. Chor. "Relocation Simulation Model for Multiple-Station Shared-Use Vehicle Systems." *Transportation Research Record* 1986, (2006): 81-88.
- Kek, Alvina G.H., Ruey L. Cheu, Q. Meng, and C.H. Fung. "Decision Support System for Vehicle Relocation Operations in Carsharing Systems." *Transportation Research Part E* 45(1), (2009): 149-158.
- Kouwenberg, Roy. "Scenario Generation and Stochastic Programming Models for Asset Liability Management." *European Journal of Operational Research* 134, (2001): 279-292.
- Nair, Rahul and Elise Miller-Hooks. "Fleet Management for Vehicle Sharing Operations." *Transportation Science* 45(4), (2011): 524-540.
- Nair, Rahul, Elise Miller-Hooks, R. Hampshire, and A. Basic. "Large-Scale Bicycle Sharing Systems: Analysis of Velib." *International Journal of Sustainable Transportation* 7(1), (2013): 85-106.
- Powell, Warren B. "A Stochastic Model of the Dynamic Vehicle Allocation Problem." *Transportation Science* 20, (1986): 117-129.
- SAS® 9.3 Macro Language Reference, SAS Institute Inc., Cary, NC, USA. 2011. <https://support.sas.com/documentation/cdl/en/mcrolref/62978/PDF/default/mcrolref.pdf>, Accessed April 2, 2014.
- Shaheen, Susan and Adam P. Cohen. "Worldwide Carsharing Growth: An International Comparison." TRB 2007 Annual Meeting CD-ROM, Paper 07-2571, National Research Council, Washington, D.C.
- Shaheen, Susan, Adam P. Cohen, and J.D. Roberts. "Carsharing in North America: Market Growth, Current Developments, and Future Potential." TRB 2006 Annual Meeting CD-ROM, Paper 06-2957, Washington D.C.
- U.S. Department of Labor. Consumer Expenditures in 2011. <http://www.bls.gov/news.release/cesan.nr0.htm>, Accessed February 2, 2014.
- Wallace, Stein. "Solving Stochastic Programs with Network Recourse." *Networks* 16, (1986): 295-317.
- Wets, Roger J.-B. "Solving Stochastic Programs with Simple Recourse." *Stochastics* 10, (1983): 219-242.
- Wollmer, Richard D. "Two State Linear Programming Under Uncertainty with 0-1 Integer First Stage Variables." *Mathematical Programming* 19, (1980): 279-288.

Zenios, Stavros A. "Dynamic Models for Fixed-income Portfolio Management Under Uncertainty." *Journal of Economic Dynamics and Control* 22, (1998): 1517-1541.

Ziemia, William. "Computational Algorithms for Convex Stochastic Programs with Simple Recourse." *Operations Research* 18, (1970): 414-431.

**Wei (David) Fan** is currently an associate professor of civil engineering at the University of North Carolina at Charlotte. He worked as an associate professor at the University of Texas at Tyler and was a senior optimization developer in the R&D Department in SAS Institute Inc. after he obtained a PhD degree in civil engineering from the University of Texas at Austin in 2004. His primary research interests include operations research, transportation network modeling and optimization, traffic simulation, traffic engineering and operations, transportation planning, and pavement management systems. He has 20 years of experience with statistical and optimization computer software development, authored over 60 journal and conference publications, and given numerous conference and invited presentations.